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AGRICULTURAL MEMORANDUM NO 843

A METHOD OF CALCULATING THE PERFORMANCE OF A SOLAR WATER HEATING SYSTEM

Introduction

During 1979 ADAS will monitor the performance of a solar water heating system in a dairy on Mr M Tory's farm at Spetisbury, Dorset. A particular system with a particular utilisation cycle will be evaluated. This is valuable by itself but the results can only be applied to other systems, in different places and with different utilisation cycles if the performance of a system can be calculated from a knowledge of its components, its operating cycle and the solar input. The object of this note is to provide a simplified mathematical description of a solar water heating system and to outline a computer model that may be used to calculate the performance of any system.

The performance of a simple system

The type of solar water heating system considered is shown in figure 1.

The notation used is:-

I	=	incident solar radiation	Wm^{-2}
A	=	area of solar collector	m^2
Ta	=	temperature of air	^oC
Tc	=	temperature of solar collector	^oC
T	=	temperature of water in storage tank	^oC
m	=	rate of flow of water through the collector	$kg\ S^{-1}$
S	=	specific heat of water	$4.2 \times 10^3\ J\ kg^{-1}\ ^oC^{-1}$
M	=	mass of water in the storage tank	kg

It is assumed that water entering the collector has the same temperature as the tank and that water leaving the collector has the same temperature as the collector.

The heat transferred from the collector to the tank each second is:

$(T_c - T)\ m \times s$ Joules 1

The energy falling on the collector during that period is:

$I \times A$ Joules 2

Therefore the efficiency is $\dfrac{(T_c - T)\ ms}{I \times A}$ 3

The efficiency of a solar collector as given by Courtney (1976) is shown in figure 2. Courtney's curve may be approximated by a linear equeation of the form

$$\text{Efficiency } E = a - b\,(T_c - T_a)/I \quad (4)$$

with a = 0.8 and b = 0.8/0.11 = 7.27

Equating 3 and 4 and rearranging gives

$$T_c\left(\frac{ms}{A} + b\right) = T\,\frac{ms}{A} + T_a \times b + a \times I \quad (5)$$

Equation 5 shows how the temperature of the water leaving the solar collector depends on the solar radiation, the collector area, the rate of mass flow through the collector, the specific heat of the circulating liquid, the temperature of the tank and the air temperature.

Rate of rise of the tank temperature

Assume that the circulating liquid has the same specific heat as water and consider the change that takes place in 1 second. If the initial tank temperature is T_o then in 1 second a mass m of water with temperature T_o is taken from the tank and replaced by a mass m with temperature T_c. The new temperature of the tank is, therefore,

$$T_1 = \frac{mT_c + (N - m)\,T_o}{M} = (T_c - T_o)\,\frac{m}{M} + T_o \qquad 6$$

Thus the rate of increase of T is $\frac{dT}{dt} = (T_c - T)\,\frac{m}{M}$ $^oC\ S^{-1}$ 7

Equation 7 may be rearranged to give

$$T_c = T + \frac{M}{m}\,\frac{dT}{dt} \qquad 8$$

Equating 5 and 8 gives

$$\frac{dT}{dt} + \frac{1}{\lambda}\,T = \frac{1}{\lambda}\,T_a + \frac{1}{\lambda} \times \frac{a}{b} \times I \qquad 9$$

Where $\frac{1}{\lambda} = \frac{m}{M}\,\left(\frac{b}{\left(\frac{ms}{A} + b\right)}\right)$ 9a

λ is the time constant of the system. For a system with 3 collectors each of 28 ft^2 (total 7.8 m^2) and a circulation rate of 6 gal/mm (0.455 kg S^{-1}) serving a 30 gallon (136.5 kg) tank and assuming b = 7.27

$$\lambda = \frac{136.5}{0.455}\left(\frac{0.455 \times 4200}{\frac{7.8}{7.27} + 7.27}\right) \triangleq 10410 \text{ second} \triangleq 2.89 \text{ hours}$$

The solution to equation (9) depends on the way that solar radiation (I) and air temperature (T_a) vary with time, the times that water is drawn off the system and the times that the pump is operating. It is most easily solved by numerical methods and a procedure for solving it is outlined later. Nevertheless, an analytic solution is possible and provides a illustration of the effect of the long time constant of the system.

Assume that solar radiation follows a cosine curve through the day reaching a maximum at local noon.

Thus $I = D \cos \frac{\pi}{\alpha}\,(t - \frac{\alpha}{2})$

Where D is the intensity of radiation at noon,
α is the number of seconds between dawn and sunset,
and t is the time in seconds measured from dawn.
Assume also that air temperature follows a cosine curve but reaches its maximum at 1500 hours local time, ie

$$T_a = \overline{T}_a + B \cos \frac{\pi}{\alpha}\,(t - \beta)$$

where $\overline{T}_a$ is the mean temperature for the day

B is the maximum temperature - mean temperature

and β is the number of seconds between dawn and time of maximum temperature.

The solution of equation 9 is then -

$$T = C\, e^{-t/\lambda} + \overline{T}_a + B \cos\theta \cos\left(\frac{\Pi t}{\alpha} - \frac{\Pi\beta}{\alpha} - \theta\right) + \frac{aD \cos\theta}{b} \sin\left(\frac{\Pi t}{\alpha} - \theta\right) \qquad 10$$

where $\cos\theta = \dfrac{1}{\sqrt{1 + \lambda^2\Pi^2/\alpha^2}}$

In equation (10) the values of $\overline{T}_a$, B, D, α and β are determined by the time of year and the meteorological conditions. The constant C may then be calculated from the initial conditions of the system. Equation (10) then specifies the temperature of water in the tank at any time between dawn and dusk if the circulating pump works continuously.

On an average April day in Dorset α = 43200 secs, β = 32400 secs, T_a = 9.1°C, B = 4.3°C and D = 437 Wm^{-2}. Assuming the tank temperature at down is 10°C and λ is 10000 secs T has been calculated from 0600 to 1500 and is shown in figure 3. It can be seen that the maximum tank temperature is about 7.5°C below the temperature that the collector would reach if it were not supplying heat to the tank and that maximum tank temperature is reached about 2.5 hours after noon. It is also apparent that the morning sun, which raises the tank temperature by 30°C is used much more efficiently than the afternoon sun which adds only 7.5°C to the tank temperature.

This is a rather unrealistic calculation because it assumes that no water is drawn off from the system but it does show the maximum temperature that could be reached by the water for a given solar input and will provide a useful check on the approximate numerical solutions of the equations that will have to be used in a computer model.

Outline of a computer model of a more complex system

The system modeled is similar to that on Mr Tory's farm at Spetisbury and its essential features are shown in figure 4. Tank 2 is normally heated by the solar collector but when the temperature of this tank reaches its thermostat setting the heat from the collector is diverted to tank 1. If the collector temperature is below the temperature of tank 1 the pump is stopped.

Given the tank temperatures at any time the computer will calculate their temperatures a short time later, taking account of the incoming radiation, the outside air temperature, heat losses from the tanks and any water drawn off the system. The procedure is described in words in figure 5. The calculations are then repeated for the next time step starting with the newly calculated tank temperatures and using new values of radiation, air temperature, etc.

A more mathematical description of the proposed program is given in Appendix I.

Reference

COURTNEY, R.G. An appraisal of solar water heating in the UK.
Building Research Establishment Current Paper CP 7/76.

Any comments or suggestions on this paper should be referred to the author.

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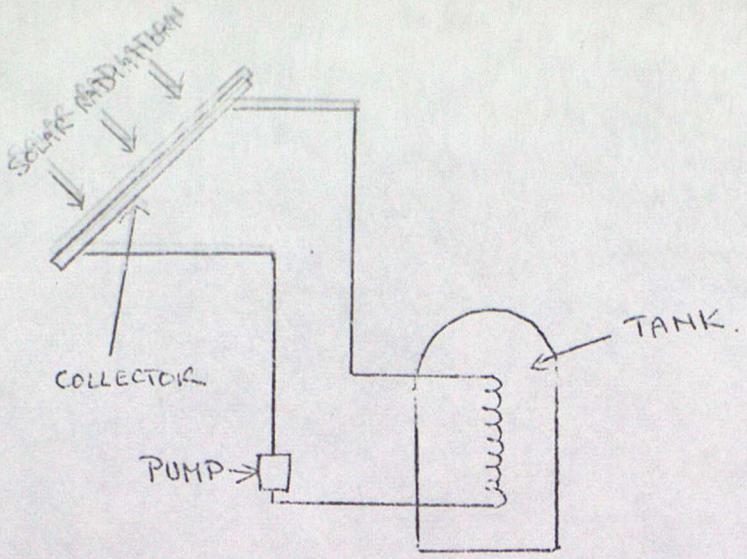
FIGURE 1



INDIRECT SOLAR WATER HEATING SYSTEM.

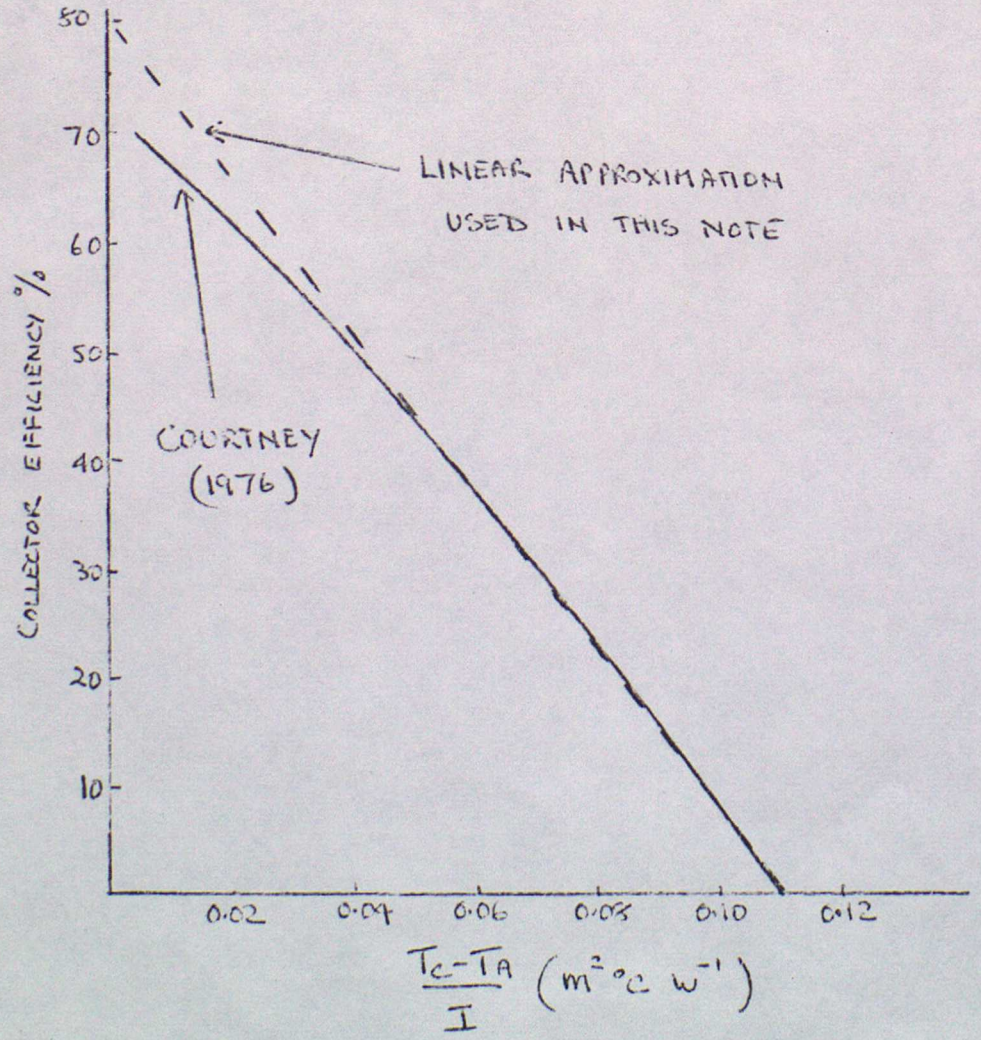


FIGURE 2. EFFICIENCY CURVE FOR A SINGLE-GLAZED COLLECTOR.

Figure 3

THE CALCULATED PERFORMANCE OF A SOLAR WATER HEATING SYSTEM ON AN 'AVERAGE' APRIL DAY IN DORSET

T_e is the equilibrium temperature of the system for the specified radiation and air temperature.

T is the calculated tank temperature.

T_a is the specified air temperature.

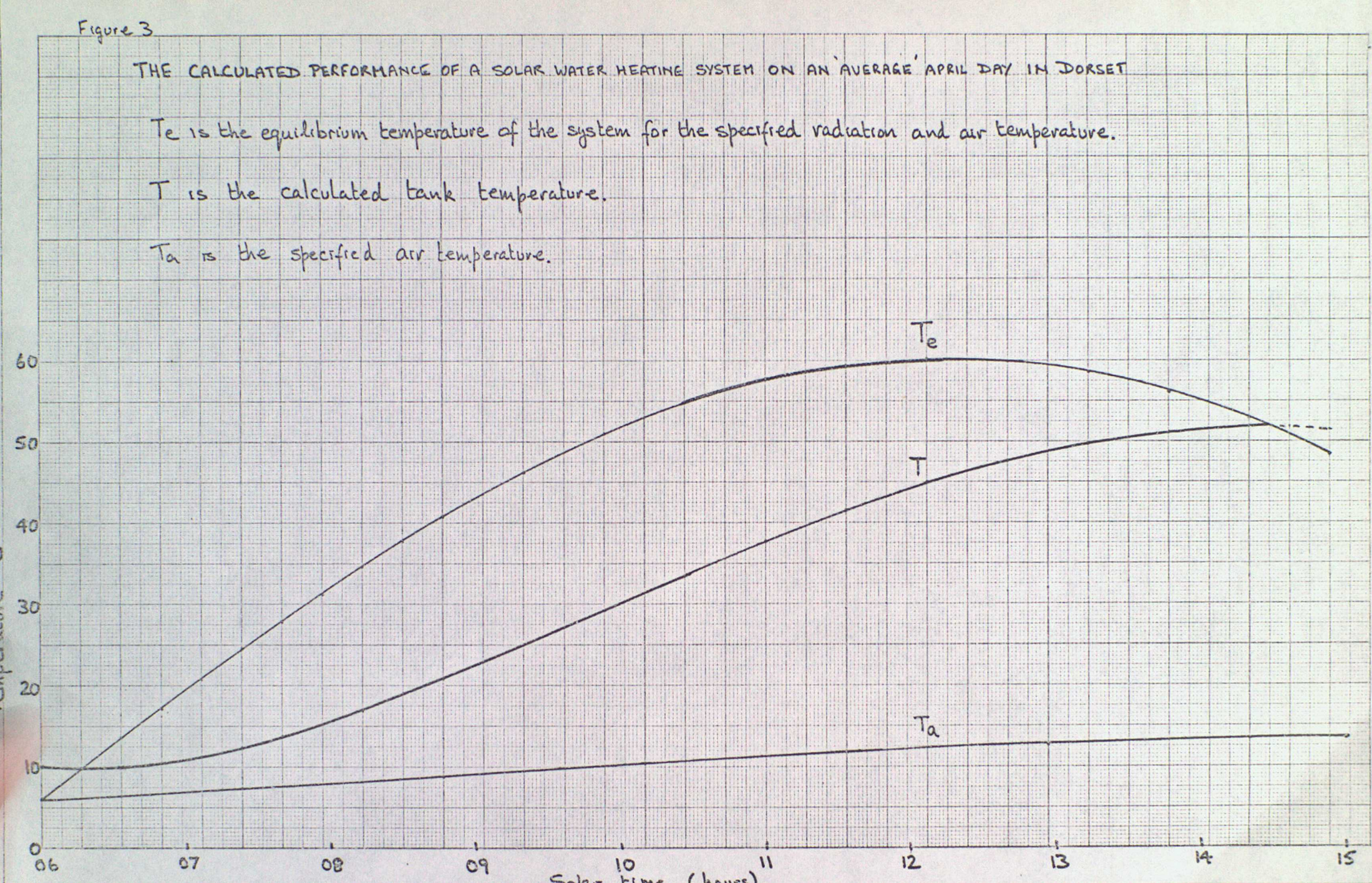


FIGURE 4

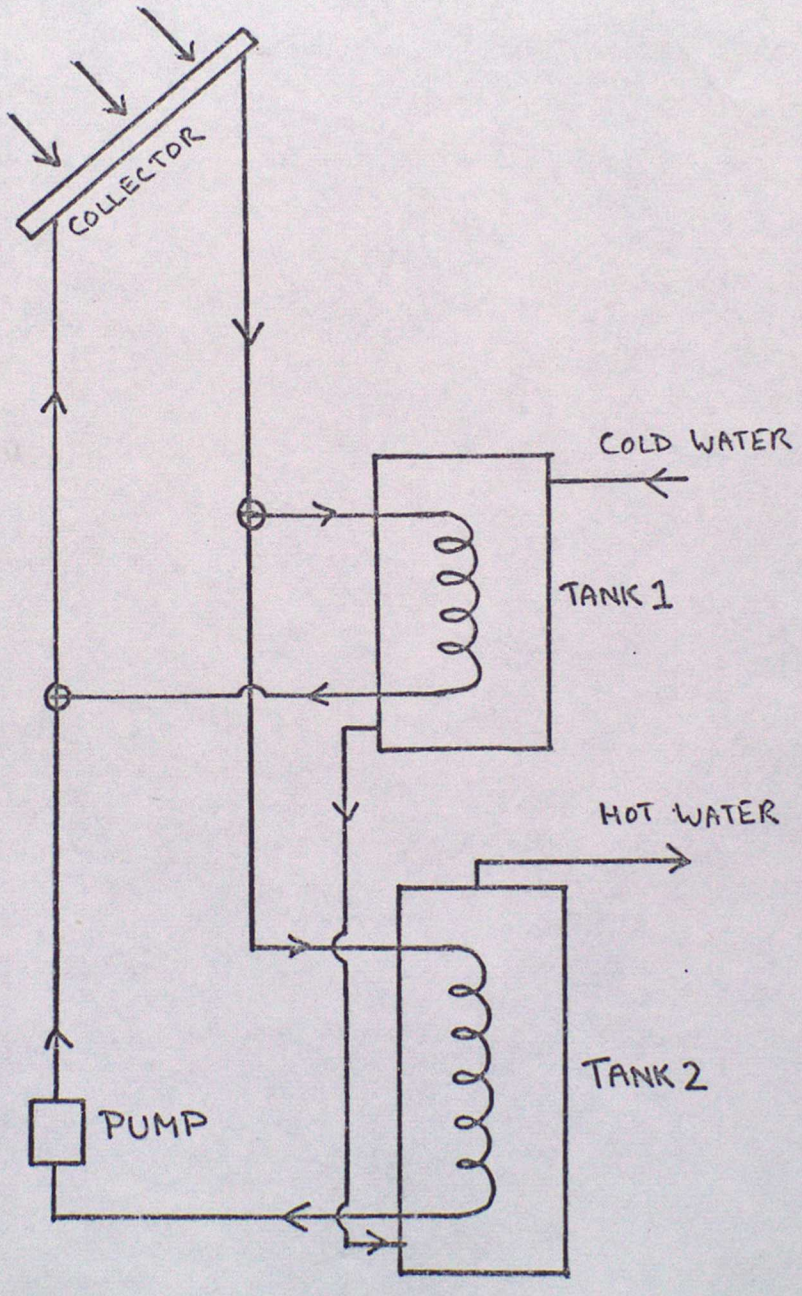
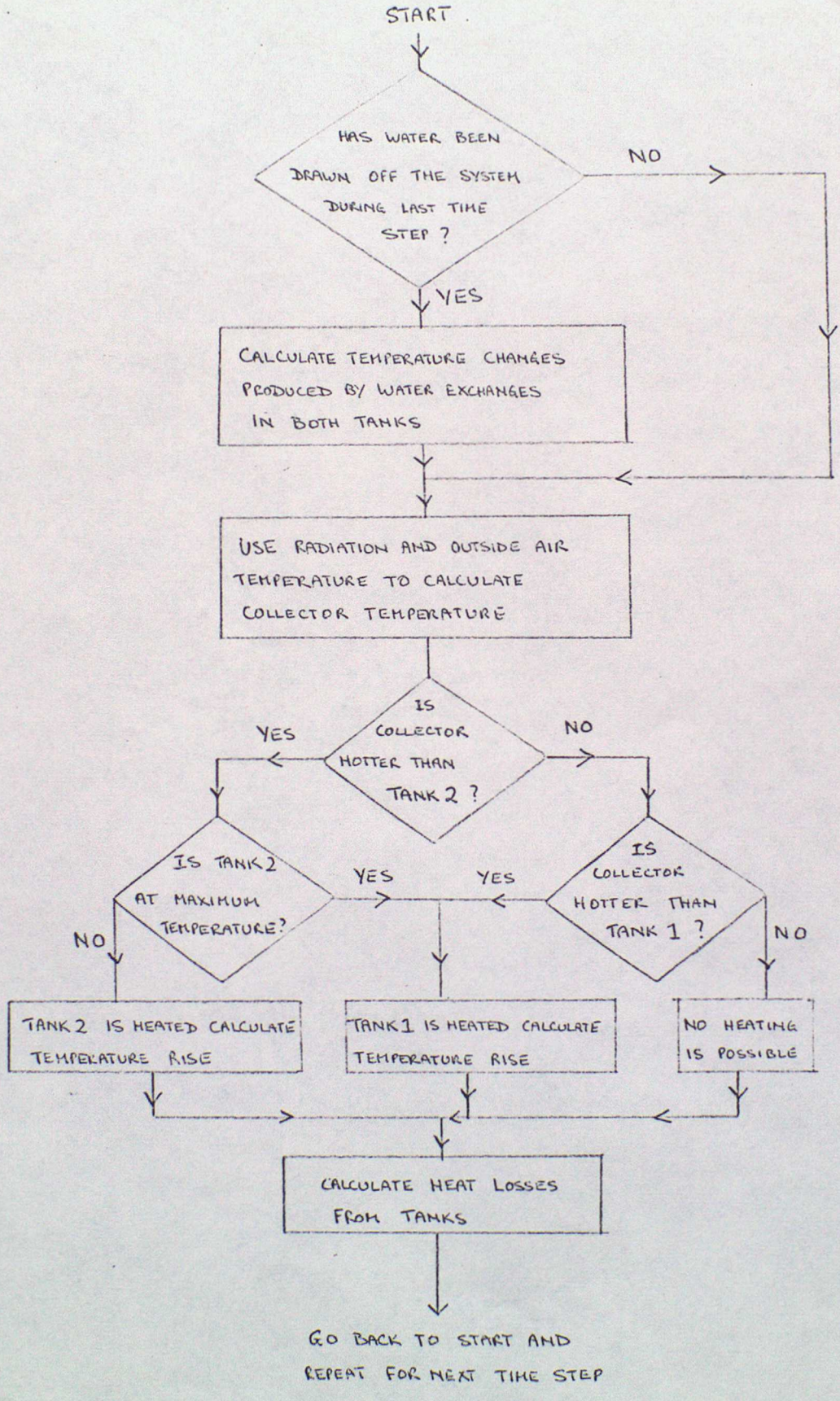


FIGURE 5


START
HAS WATER BEEN DRAWN OFF THE SYSTEM DURING LAST TIME STEP ?
NO
YES
CALCULATE TEMPERATURE CHANGES PRODUCED BY WATER EXCHANGES IN BOTH TANKS
USE RADIATION AND OUTSIDE AIR TEMPERATURE TO CALCULATE COLLECTOR TEMPERATURE
IS COLLECTOR HOTTER THAN TANK 2 ?
YES
NO
IS TANK 2 AT MAXIMUM TEMPERATURE?
YES
NO
IS COLLECTOR HOTTER THAN TANK 1 ?
YES
NO
TANK 2 IS HEATED CALCULATE TEMPERATURE RISE
TANK 1 IS HEATED CALCULATE TEMPERATURE RISE
NO HEATING IS POSSIBLE
CALCULATE HEAT LOSSES FROM TANKS
GO BACK TO START AND REPEAT FOR NEXT TIME STEP

APPENDIX I

Outline of a computer model to simulate a solar water heating system

A diagram of the system modelled is shown in figure 4.

VARIABLES

TC	=	temperature of collector (^{o}C)
TCE	=	equilibrium temperature of collector when no heat is extracted from it (^{o}C)
TWIN	=	temperature of water taken into tank 1 (^{o}C)
T1	=	temperature of water in tank 1 (^{o}C)
T2	=	temperature of water in tank 2 (^{o}C)
TAC	=	temperature of air around collector (^{o}C)
TAD1	=	temperature of air in dairy around tank 1 (^{o}C)
TAD2	=	temperature of air in dairy around tank 2 (^{o}C)
TMAX2	=	thermostat setting for tank 2 (^{o}C)
MC	=	mass of water circulating through the collector each sec ($kg.s^{-1}$)
M1	=	mass of water in tank 2 (kg)
M2	=	mass of water in tank 2 (kg)
MOUT	=	mass of water drawn off the system during 1 timestep (kg)
RAD	=	solar radiation (Wm^{-2})
A	=	area of solar collector (m^{2})
S	=	specific heat of water = 4200 $J.kg^{-1}\,{}^{o}C^{-1}$
EMAX	=	maximum efficiency of collector
SE	=	slope of the collector efficiency curve

NOTE: Collector efficiency = EMAX - SE (TC - TA)/RAD cf equation 4

LAG1	=	time constant when tank 1 is heated. (sec)
LAG2	=	time constant when tank 2 is heated. (sec)
C1	=	time constant for cooling of tank 1. (sec)
C2	=	time constant for cooling of tank 2. (sec)
DIFF	=	temperature difference required between collector and tank before pump starts (^{o}C)
DTIME	=	length of a time step (sec)

Derivation of equations used in the model

Calculation of time constants for heating

From equation (9a)

$$LAG1 = \frac{M1}{MC} \times \frac{(MC \times S}{A} + SE)/SE \qquad (A1)$$

$$LAG2 = \frac{M2}{MC} \times \frac{(MC \times S}{A} + SE)/SE \qquad (A2)$$

Calculation of temperature rise in a tank during one time step

If we consider the small time interval δt the temperature rise ΔT in a tank during that time can be estimated from equation (9) using the finile difference approximation.

$$\Delta T + \frac{\delta t}{\lambda} T = \frac{\delta t}{\lambda} T_a + \frac{\delta t}{\lambda} \times \frac{a}{b} \times I \qquad (A3)$$

Therefore, if T_n is the tank temperature after n time steps the tank temperature after n + 1 time steps is given by.

$$T_{n+1} = T_n \left(1 - \frac{\delta t}{\lambda}\right) + \frac{\delta t}{\lambda} T_a + \frac{\delta t}{\lambda} \times \frac{a}{b} \times I \qquad (A4)$$

Calculation of temperature loss from a tank during one time step

Consider tank 1

$$\frac{d\,T1}{dt} = - \frac{1}{C1} (T1 - TAD1) \qquad (A5)$$

or

$$\Delta T1 \triangleq - \frac{\delta t}{C1} (T1 - TAD1) \qquad (A6)$$

Therefore, the tank temperature after n + 1 time steps is related to that after n time steps by the equation.

$$T1_{n+1} = T1_n \left(1 = \frac{\delta t}{C1}\right) + \frac{\delta t}{C1} \times TAD1 \qquad (A7)$$

Similarly

$$T2_{n+1} = T2_n \left(1 - \frac{\delta t}{C2}\right) + \frac{\delta t}{C2} \times TAD2 \qquad (A8)$$

Temperature changes when water is drawn off from the system

When hot water is extracted from tank 2 it is replaced by water from tank 1 and an equal mass of water is drawn into tank 1.

If $T1_n$ is the temperature in tank 1 before the exchange of water the temperature after the exchange is

$$T1_{n+1} = \frac{T1_n \times (M1 - MOUT) + MOUT \times TWIN}{M1} \qquad (A9)$$

The temperature of tank 2 after the exchange is

$$T2_{n+1} = \frac{T2_n \times (M2 - MOUT) + MOUT \times T1_n}{M2} \qquad (A10)$$

Calculation of collector equilibrium temperature

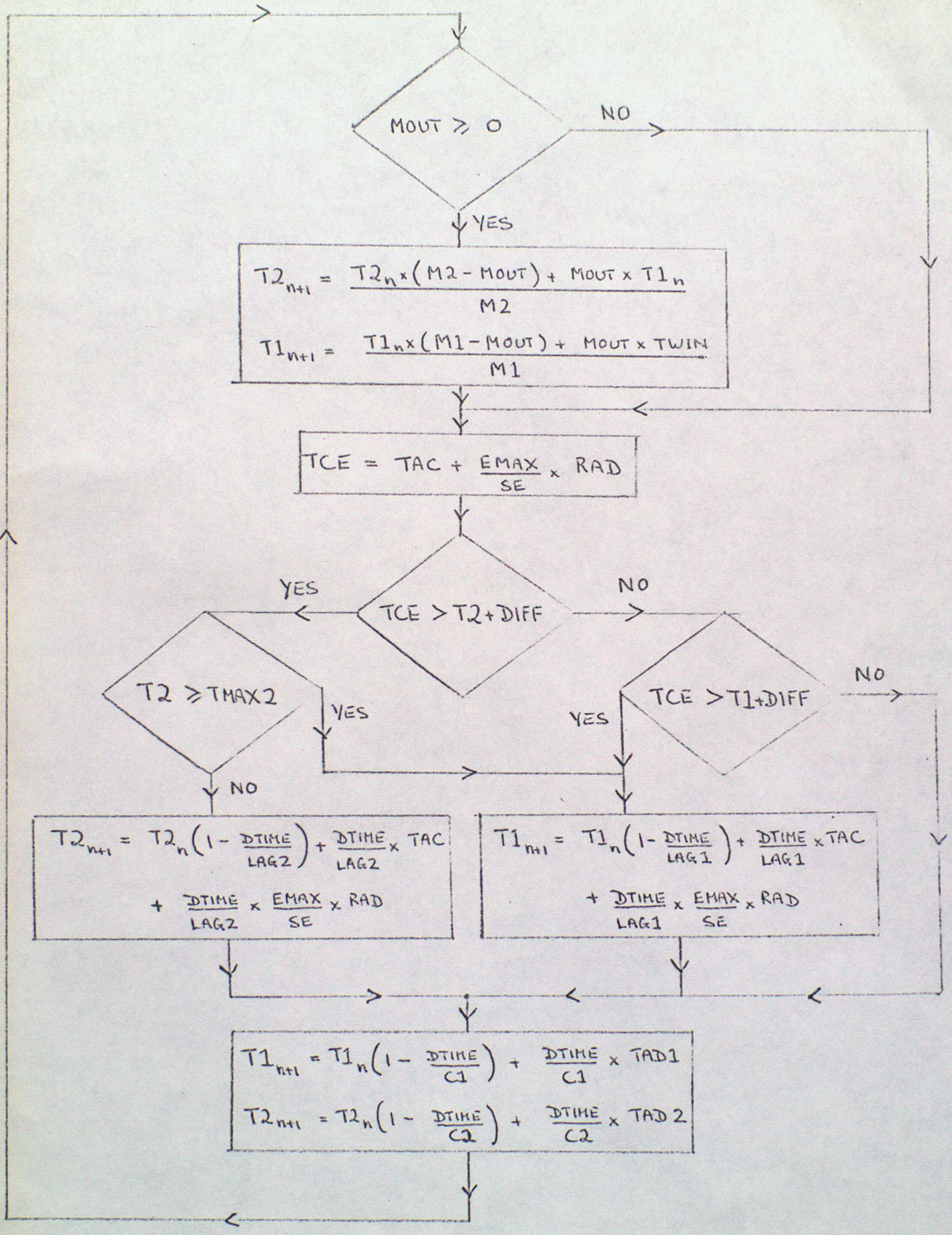
This is obtained by putting m = o (ie no circulation) in equation (5).

$$\text{Thus } TCE = TAC + \frac{EMAX}{SE} \times RAD \qquad \text{(A11)}$$

Flow diagram

The way the above equations are linked to calculate the changes in tank temperatures during a single time step of the program is shown in figure A1.

FIGURE A1


MOUT ≥ 0
NO
YES
$T2_{n+1} = \frac{T2_n \times (M2 - MOUT) + MOUT \times T1_n}{M2}$
$T1_{n+1} = \frac{T1_n \times (M1 - MOUT) + MOUT \times TWIN}{M1}$
$TCE = TAC + \frac{EMAX}{SE} \times RAD$
TCE > T2+DIFF
YES
NO
T2 ≥ TMAX2
YES
NO
TCE > T1+DIFF
YES
NO
$T2_{n+1} = T2_n\left(1 - \frac{DTIME}{LAG2}\right) + \frac{DTIME}{LAG2} \times TAC + \frac{DTIME}{LAG2} \times \frac{EMAX}{SE} \times RAD$
$T1_{n+1} = T1_n\left(1 - \frac{DTIME}{LAG1}\right) + \frac{DTIME}{LAG1} \times TAC + \frac{DTIME}{LAG1} \times \frac{EMAX}{SE} \times RAD$
$T1_{n+1} = T1_n\left(1 - \frac{DTIME}{C1}\right) + \frac{DTIME}{C1} \times TAD1$
$T2_{n+1} = T2_n\left(1 - \frac{DTIME}{C2}\right) + \frac{DTIME}{C2} \times TAD2$